Brainiacs Math Olympiad Preliminary Round Sample Exam Paper

Category V – grades 11 and 12

Q1.

Find m if $P(x) = x^4 + 5x^3 + mx + 1$ is divisible by $x + 2$.

A) 4

B) -8

C) -11.5

D) 9.5

Q2.

What is the sum of the coefficients of $P(4x - 7)$ if $4 \bullet P(x) + 2 \bullet P(-x) = 3x^2 + 2x - 3$?

A) 4

B) 3

C) 2

D) 1

Q3.

Factorize $x^4 + x^2 + 1$.

A) $(x^2 + 1)(x^2 - 1)$

B) $x^2(x^2 + 1) + 1$

C) $(x^2 + x + 1)(x^2 - x + 1)$

D) $(x^2 + x + 1)^2$

Q4.

Decompose the expression $\frac{7x - 20}{x^2 - 4x - 32}$ into partial sums.

A) $\frac{5}{x - 8} + \frac{2}{x + 4}$

B) $\frac{4}{x - 8} + \frac{3}{x + 4}$

C) $\frac{3}{x - 8} + \frac{4}{x + 4}$

D) $\frac{2}{x-8}+\frac{5}{x+4}$

Q5.

Find $x_1^2+\frac{1}{x_1^2}$, if x_1 is the root of the equation $\left(x+\frac{1}{x}\right)^2-6\left(x+\frac{1}{x}\right)+9=0.$

A) $\frac{3+\sqrt{5}}{2}$

B) 5

C) $\frac{3-\sqrt{5}}{2}$

D) 7

Q6.

Solve the equation $x^3+3x^2-2x-2=0.$

A) $x_1=1, x_2=-2+\sqrt{6}, x_3=-2-\sqrt{6}$

B) $x_1=1, x_2=-1+\sqrt{3}, x_3=-1-\sqrt{3}$

C) $x_1=1, x_2=-2+\sqrt{5}, x_3=-2-\sqrt{5}$

D) $x_1=1, x_2=-2+\sqrt{3}, x_3=-2-\sqrt{3}$

Q7.

Solve $\begin{cases}\sqrt{\frac{x+5}{2x-1}}>2\\ \frac{4-3x}{\sqrt{40-3x}}<\sqrt{5}\end{cases}$

A) $\left(1;\frac{7}{5}\right)$

B) $\left(\frac{1}{2};\frac{9}{7}\right)$

C) $\left(1;\frac{13}{6}\right)$

D) $\left(\frac{1}{4};\frac{40}{3}\right)$

Q8.

Find the value of x.

$$\frac{2-4\log_{12}2}{\log_{12}(x+2)}-1=\frac{\log_6(8-x)}{\log_6(x+2)}$$

A) 8

B) 7

C) 5

D) -1

Q9.

Find the all set of (x,y).

$$\begin{cases} x^{\log_8 y}+y^{\log_8 x}=4 \\ \log_4 x-\log_4 y=1 \end{cases}$$

A) $(1,\frac{1}{4})$

B) $(1,\frac{1}{4}),\ (\frac{1}{2},\frac{1}{8})$

C) $\left(\frac{1}{2},\frac{1}{8}\right),(8,2)$

D) $(8,2)$

Q10.

The 15th term of an arithmetic progression is 24, and the sum of the first 15 terms is 570. The sum of the first N terms of this arithmetic progression is 0. Determine the value of N.

A) 52

B) 53

C) 49

D) 47

Q11.

The sum of two terms of a geometric progression is 10, and the sum to infinity of the geometric progression is 18. Determine the two possible values of the common ratio, r.

A) $3 \; and \; -3$

B) $\frac{2}{3} \; and \; -\frac{2}{3}$

C) $\frac{3}{2} \; and \; -\frac{3}{2}$

D) $2 \; and \; -2$

Q12.

In how many ways can 3 Pakistani, 5 Indian, and 4 Chinese diplomats be seated around a circular table if diplomats from the same country must sit next to each other?

A) 60

B) 2880

C) 5760

D) 11520

Q13.

Classes 10 A and 10 B have 12 and 18 students, respectively. A basketball team consisting of 5 players will be formed by selecting 2 students from Class 10 A and 3 students from Class 10 B. How many different teams can be formed?

A) $1\ 296$

B) $13\ 464$

C) $26\ 928$

D) $53\ 856$

Q14.

There are 4 permanent members and 9 elected members on the company's board of directors. For a decision to be approved, there must be at least 8 votes in favor, and all permanent members must vote in favor. In how many ways can a decision be passed by the board, assuming that all members participate in the voting?

A) 164

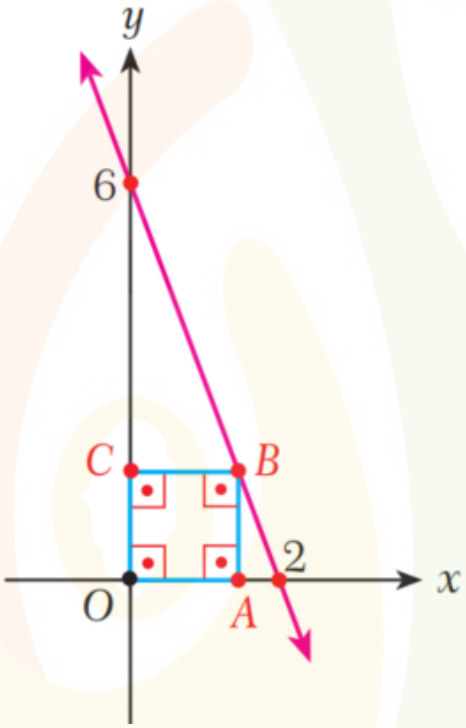
B) 382

C) 424

D) 51215

Q15.

In the figure, OABC is a square. Find the area of the square.



A) 1.96

B) 3.24

C) 1.69

D) 2.25

Q16.

The Australian 50-cent coin has the shape of a regular dodecagon, which is a polygon with 12 sides.

Eight of these 50-cent coins will fit exactly on an Australian $10 note as shown. What fraction of the $10 note is not covered by the coins?

A) $\frac{1}{3}$

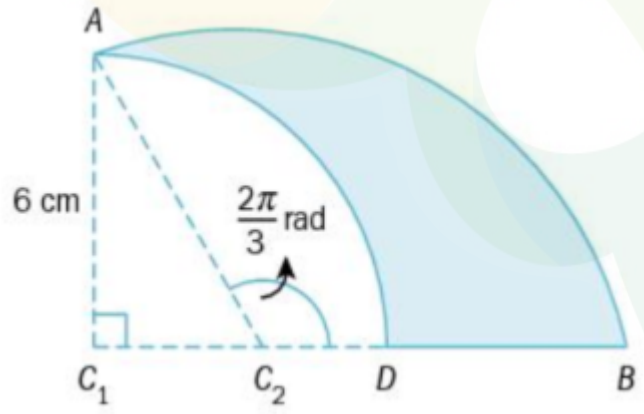
B) $\frac{3}{4}$

C) $\frac{1}{4}$

D) $\frac{2}{3}$

Q17.

Arc AD is a part of a circle with center C_1 and radius of 6 cm. Arc AB is a part of a circle with center C_2. The measure of angle AC_2D is $\frac{2\pi}{3}$ radians.



Calculate the area of the shaded region ABD.

A) $12\ cm^2$

B) $\left(7\pi + 6\sqrt{3}\right)cm^2$

C) $\left(6 - 2\sqrt{3}\right)cm^2$

D) $\left(2\pi - 3\sqrt{3}\right)cm^2$

Q18.

Find the simplest form of $\dfrac{sin^4x + cos^4x - 1}{sin^6x + cos^6x - 1}$.

A) 1

B) $\dfrac{2}{3}$

C) $\dfrac{2}{sinx}$

D) $tanx$

Q19.

An arrow is shot upward on a planet. Its height (in meters) after t seconds is given by equation $h(t) = 60t - 0.6t^2$. At what time will the arrow reach its maximum height? What will be the velocity of the arrow when it hits the ground?

A) $50s\ and\ 60m/s$

B) $40s\ and\ 50m/s$

C) $50s\ and\ 72m/s$

D) $40s\ and\ 62m/s$

Q20.

A manufacturer has an order to produce cylindrical cans with a volume of 500 cm³. Determine the radius of the cans that will minimize the cost of the metal used in their production.

A) $\sqrt[3]{\dfrac{250}{\pi}}$

B) $\left(\dfrac{\pi}{25}\right)^3$

C) $\sqrt[3]{25\pi}$

D) $\sqrt[3]{\dfrac{\pi}{25}}$