Brainiacs Math Olympiad Preliminary Round Sample Exam Paper

Category IV – grades 9 and 10

Q1.

Simplify the expression

$$\left(\frac{a^{-2} \cdot b^{-1}}{b^3}\right)^3 \times \left(\frac{a^2 \cdot b^2}{b^{-2}}\right)^2 \times \left(\frac{b^{-1}}{a^2 \cdot b^{-4}}\right)^{-2}$$

A. $\dfrac{a^2}{b^6}$

B. $\dfrac{a^2}{b^{10}}$

C. $\dfrac{a^{-2}}{b^{-10}}$

D. $\dfrac{a^{-2}}{b^6}$

Q2.

Find the value of n if $= 3^{4n+2}$ and $81m^3 = 3^{7n}$.

A. 2

B. 4

C. -2

D. -4

Q3.

Perform the operation.

$$\frac{1}{\sqrt{2}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{8}} + \dots + \frac{1}{\sqrt{252}+\sqrt{254}} + \frac{1}{\sqrt{254}+\sqrt{256}}$$

A. $\dfrac{\sqrt{2}-16}{2}$

B. $\dfrac{16-\sqrt{2}}{2}$

C. $16-\sqrt{2}$

D. $\sqrt{2}-16$

Q4.

Find the value of x^2.

$$\sqrt[3]{x+2} - \sqrt[3]{x-2} = 4.$$

A. -121

B. -74

C. -53

D. -46

Q5.

Two squares have side lengths of (x + 6) cm and (2x + 1) cm, respectively. The sum of their areas is 697 cm². Find the area of the larger square.

A. 484 cm²

B. 441 cm²

C. 400 cm²

D. 361 cm²

Q6.

Find $(x_1)^2 + \dfrac{1}{(x_1)^2}$, if x_1 is the root of the equation $\left(x + \dfrac{1}{x}\right)^2 - 6\left(x + \dfrac{1}{x}\right) + 9 = 0$

A. 13

B. 9

C. 7

D. 4

Q7.

Solve the equation

$$\sqrt{3x^2 + 5x + 8} - \sqrt{3x^2 + 5x + 1} = 1$$

A. 1

B. $-\dfrac{8}{3}$

C. $3 \ and - \dfrac{8}{3}$

Q8.

A farmer picks 120 tomatoes on the first day of the harvest, and each subsequent day, he picks 40 more tomatoes than the previous day. How many days will it take for the farmer to pick a total of 3,000 tomatoes?

A. 21 days

B. 15 days

C. 10 days

D. 7 days

Q9.

The sum of the third and fifth terms of an arithmetic sequence is 20, and the product of the fourth term and the sixth term is 200. Find the third term of this sequence.

A. 5

B. 9

C. 12

D. 14

Q10.

A ball is dropped from a height of 81 cm. Each time it bounces, it returns to $\dfrac{2}{3}$ of its previous height. What is the total distance the ball has traveled in the air when it hits the ground for the fifth time?
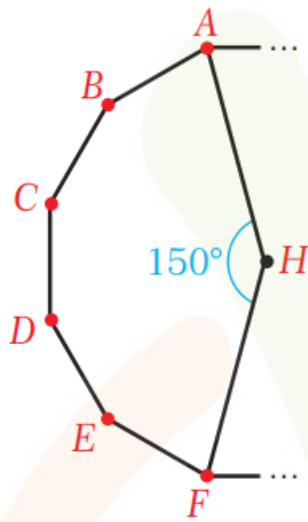
A. 341 cm

B. 368 cm

C. 417 cm

D. 523 cm

Q11.

In the figure, *ABCDEF*... is a regular polygon

and line segments *AH* and *FH* bisect ∠A and ∠F respectively. If $m(\angle AHF)$ = 150°, find the number of sides of the polygon.

A. 18

B. 15

C. 14

D. 12

Q12.
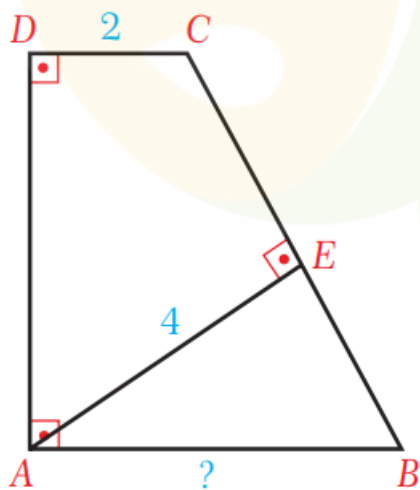
In the right trapezoid *ABCD*, *AB* is parallel to *DC* and *AE* ⊥ *BC*.

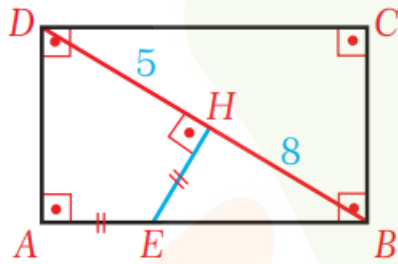Given *AB* = *BC*, *DC* = 2 cm and *AE* = 4 cm, find the length of *AB*.

A. 12 cm

B. 9 cm

C. 7 cm

D. 5 cm

Q13.

In the figure, *ABCD* is a rectangle. Given *EH* ⊥ *BD*, *AE* = *EH*, *DH* = 5 cm and *HB* = 8 cm, find perimeter *P(AEHD)*.

A. $\dfrac{50}{3}$

B. 25

C. 16

D. $\dfrac{32}{3}$

Q14.

Find the value of x. $3^{2x} + \dfrac{36}{3^{2x}} - \left(3^x + \dfrac{6}{3^x}\right) - 8 = 0$

A. 1

B. $\log_2 3$

C. $1 \ and \ \log_3 2$

D. $\log_3 2$

Q15.

Solve the equation. $\log_{3x-2}(2x^2) = \log_{3x-2}(x+1)$

A. 1

B. 2

C. 3

D. 4

Q16.

Solve the system equation

$$\begin{cases} \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{2}{15} \\ \log_3 x + \log_3 y = 1 + \log_3 5 \end{cases}$$

A. (1,4)

B. (2,3)

C. (3,5)

D. (3,9)

Q17.

Six men and seven women apply for the positions of manager and assistant manager in a company. Each name is written on a card, and all the cards are placed in a box. Two names are then selected in succession from the box at random, without replacement. What is the probability that both people selected are men?

A. $\dfrac{6}{13}$

B. $\dfrac{5}{12}$

C. $\dfrac{5}{26}$

D. $\dfrac{6}{26}$

Q18.

A coin is tossed, and a die is rolled. What is the probability that the coin shows heads or the die shows an even number?

A. $\dfrac{1}{4}$

B. $\dfrac{3}{4}$

C. 1

D. $\dfrac{1}{2}$

Q19

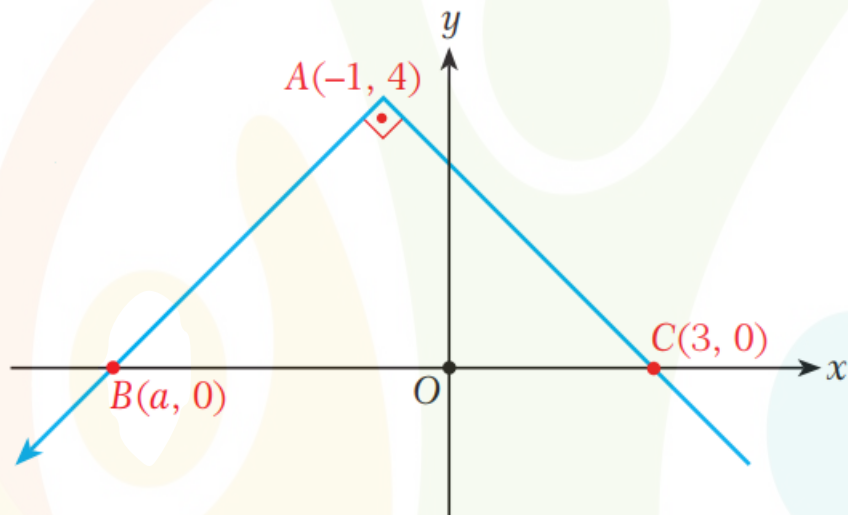Solve the equation $sin^4x + cos^4x = sin x \bullet cos x$

A. $\dfrac{\pi}{4} + k\pi, \; k \epsilon Z$

B. $\dfrac{\pi}{6} + k\pi, \; k \epsilon Z$

C. $\dfrac{3\pi}{4} + 2k\pi,\ k\epsilon Z$

D. $\dfrac{\pi k}{4},\ k\epsilon Z$

Q20.

In the figure, find the value of a.

$A(-1, 4)$

$B(a, 0)$

O

$C(3, 0)$

y

x

A. -3.5

B. -5

C. -6

D. -6.4