Brainiacs Math Olympiad Preliminary Round Sample Exam Paper 3

Category V – grade 11 and 12

Q1.

Find the sum of the values of x that satisfy the equation below.

$$2^{\log_x(x^2-6x+9)} = 3^{2(\log_x\sqrt{x})-1}$$

A) 4
B) 5
C) 6
D) 7

Q2.

With i being the imaginary unit, which of the following is equivalent to $\sqrt{-36} \bullet \sqrt{-25}$?

A) 30
B) -30
C) $30i$
D) $-30i$

Q3.

Let a, b, c, and d be non-negative integers, with $3^a + 3^b + 3^c + 3^d = 2512$. Find $a + b + c + d$.

A) 12
B) 14
C) 16
D) 18

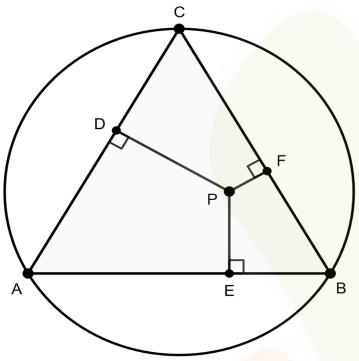
Q4.

The sum of the odd-order terms of an infinite geometric progression is 8 and the sum of the even-order terms is 4. What is the first term of this progression?

A) 6
B) 7
C) 8
D) 9

Q5.

In the figure below, triangle ABC is equilateral and is inscribed in a circle of radius 8. Knowing that $FP:EP:DP = 1:2:3$, find the area of triangle DEF.



A) $11\sqrt{3}$
B) $9\sqrt{3}$
C) $7\sqrt{3}$
D) $5\sqrt{3}$

Q6.

What is the surface area, in square centimeters, of a sphere that is inscribed in a cube whose diagonal measures 12 centimeters? Use $\pi = 3$.

A) 144
B) 216
C) 432
D) 576

Q7.

Find the last two digits of the number below.

$$2023^{2024}$$

A) 11
B) 21
C) 31
D) 41

Q8.

Find the value of the sum below, considering that $\lceil x \rceil$ represents the smallest integer not smaller than x. For example: $\lceil 13.05 \rceil = 14$.

$$\left\lceil \sqrt{1} \right\rceil + \left\lceil \sqrt{2} \right\rceil + \left\lceil \sqrt{3} \right\rceil + \ldots + \left\lceil \sqrt{99} \right\rceil + \left\lceil \sqrt{100} \right\rceil$$

A) 500
B) 625
C) 680
D) 715

Q9.

Ann and John are playing a game in which they take turns removing, without replacement, balls from a box. This box has 3 green balls and 7 white balls, but it is not possible to see the color of the balls before removing them. The winner is the one who removes the first green ball. If Ann was the first to remove a ball, what is the probability that she will win the game?

A) $\frac{3}{10}$

B) $\frac{1}{2}$

C) $\frac{7}{12}$

D) $\frac{3}{4}$

Q10.

What is the sum of the digits of the coefficient of x^{23} in the expansion of the polynomial below?

$$(1 + x^3 + x^5)^{10}$$

A) 3

B) 5

C) 7

D) 9

Q11.

Look at the equation below. There are two possible values for $\frac{24y}{x}$. Find the sum of these values.

$$35x^2 + xy - 12y^2 = 0$$

A) 1

B) 2

C) 3

D) 4

Q12.

A certain ice cream shop in New York has 4 flavors of ice cream, which are sold in cups with 5 ice cream scoops. How many ways can you buy a cup of ice cream at this ice cream shop?

A) 14

B) 56

C) 256

D) 1024

Q13.

Knowing that the polynomial $p(x) = x^3 - 26x^2 + 216x - 576$ has 3 positive real roots, calculate the harmonic mean between these roots.

a) 6

b) 7

c) 8

d) 9

Q14.

Six friends will organize a Secret Santa, a game in which they write their names on pieces of paper, put them into an urn, and each participant then randomly removes a piece of paper without telling the others. The fun is in the fact that each person must buy a gift for the friend who is drawn. For the game to work properly, it is necessary that no person draws his or her own name, and the probability of this happening is given by the irreducible fraction $\frac{a}{b}$. So, find the value of $a + b$.

a) 197

b) 203

c) 205

d) 211

Q15.

Find the largest three-digit number that leaves remainder 1 on division by 3, remainder 2 on division by 4, and remainder 3 on division by 5.

a) 958

b) 798

c) 898

d) 878

Q16.

Let a, b, x, and y be real numbers that satisfy the system of equations below. Accordingly, find the value of $a\,x^4 + b\,y^4$.

$$\begin{cases} a + b = 24 \\ ax + by = 97 \end{cases}$$

$$\begin{cases} a\,x^2 + b\,y^2 = 241 \\ a\,x^3 + b\,y^3 = 823 \end{cases}$$

a) 2464

b) 2357

c) 2269

d) 2174

Q17.

Knowing that $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 2 \bullet 1$, find the number of zeros at the end of $2025!$

a) 505

b) 503

c) 498

d) 493

Q18.

If $x + \frac{1}{x} = 5$, what is the value of $x^5 + \frac{1}{x^5}$?

a) 2025

b) 2525

c) 2725

d) 3125

Q19.

Let m and n be natural numbers. Knowing that $3^m - 3^n = 1944$, calculate the value of 3^{m-n}.

A) 3

B) 9

C) 27

D) 81

Q20.

Find the value of the expression below.

$$\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}}$$

A) 4

B) 3

C) 2

D) 1