Brainiacs Math Olympiad Preliminary Round Sample Exam Paper 3

Category IV – grade 9 and 10

Q1.

From the information below, find the value of $f(2)$.

$$f\Big(f\Big(f\big(f(f(x))\big)\Big)\Big) = x + 15$$

A) 1
B) 3
C) 5
D) 7

Q2.
What is the value of the sine of 75°?

A) $\dfrac{\sqrt{2}+\sqrt{3}}{2}$

B) $\dfrac{1+\sqrt{2}}{2}$

C) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

D) $\dfrac{1+\sqrt{3}}{2\sqrt{2}}$

Q3.
Let m and n be natural numbers. Knowing that $3^m - 3^n = 1944$, calculate the value of 3^{m-n}.

A) 3
B) 9
C) 27
D) 81

Q4.
Andrew needs to climb an 8-step ladder. Since he has large legs, he can do this by climbing 1, 2, or 3 steps at a time. How many ways can Andrew get to the top of the ladder?

A) 81
B) 68
C) 31
D) 8

Q5.

If $x + \frac{1}{x} = 5$, what is the value of $x^4 + \frac{1}{x^4}$?

A) 525
B) 527
C) 529
D) 531

Q6.
The diagonals of a parallelogram measure 14 and 10 centimeters and form an angle of 150°. Calculate the area of this parallelogram in square centimeters.

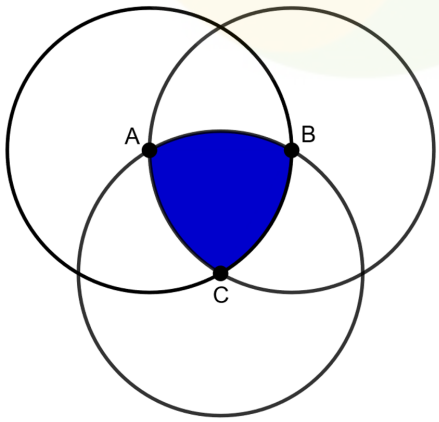
A) 35
B) 70
C) 140
D) 280

Q7.
Find the value of the sum below, considering that $\lceil x \rceil$ represents the smallest integer not smaller than x. For example: $\lceil 13.05 \rceil = 14$.

$$\left\lceil \sqrt{1} \right\rceil + \left\lceil \sqrt{2} \right\rceil + \left\lceil \sqrt{3} \right\rceil + \ldots + \left\lceil \sqrt{99} \right\rceil + \left\lceil \sqrt{100} \right\rceil$$

A) 500
B) 625
C) 680
D) 715

Q8.
In the figure below, three identical circles with a diameter of 20 centimeters and centers A, B, and C are arranged so that each circle passes through the center of the other two. What is the area of the blue region in square centimeters? Consider $\pi = 3$ and $\sqrt{3} = 1.7$.



A) 65
B) 130
C) 175
D) 260

Q9.

Ann will take a test with 25 multiple-choice questions. The scoring of the test works as follows: all students start with 25 bonus points, get 4 points for a correct answer, lose 1 point for an incorrect answer, and get 0 points for an answer left blank. If Ann had a score of 94, how many questions did she leave blank?

A) 1
B) 2
C) 3
D) 4

Q10.

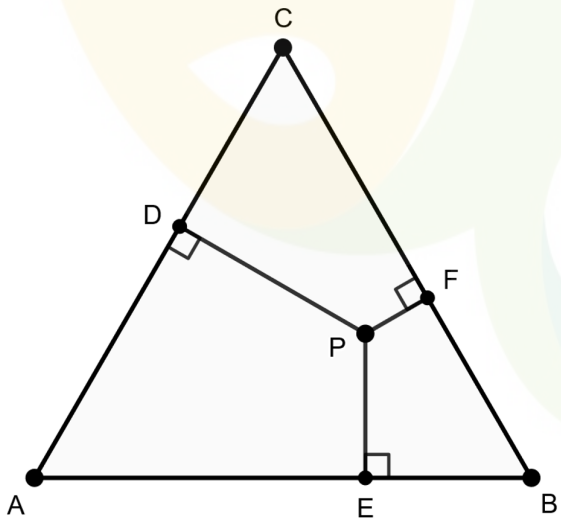
Determine the number of integer solutions of the equation below.

$$(x^2 - x + 1)^{(2x^2-3x-2)} = 1$$

A) 1
B) 2
C) 3
D) 4

Q11.

In the figure below, triangle ABC is equilateral, $DP = 3$, $EP = 2$, $FP = 1$ and the angles marked are right angles. Find the height of the triangle ABC.



A) 4
B) 6
C) $4\sqrt{3}$
D) $6\sqrt{3}$

Q12.
A certain ice cream shop in New York has 4 flavors of ice cream, which are sold in cups with 5 ice cream scoops. How many ways can you buy a cup of ice cream at this ice cream shop?

A) 14
B) 56
C) 256
D) 1024

Q13.
What is the surface area, in square centimeters, of a sphere that is inscribed in a cube whose diagonal measures 12 centimeters? Use $\pi = 3$.

A) 144
B) 216
C) 432
D) 576

Q14.
Let g be a second-degree function such that $g(1) = 12$, $g(2) = 5$, and $g(3) = 0$. What is the value of $g(4)$?

A) -3
B) 1
C) 3
D) 5

Q15.
Look at the equation below. There are two possible values for $\frac{24y}{x}$. Find the sum of these values.

$$35x^2 + xy - 12y^2 = 0$$

A) 1
B) 2
C) 3
D) 4

Q16.
Knowing that $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 2 \bullet 1$, find the number of zeros at the end of $2025!$.

a) 505
b) 503
c) 498
d) 493

Q17.

Four friends will organize a Secret Santa, a game in which they write their names on pieces of paper, put them into an urn, and each participant then randomly removes a piece of paper without telling the

others. The fun is in the fact that each person must buy a gift for the friend who is drawn. For the game to work properly, it is necessary that no person draws his or her own name, and the probability of this happening is given by the irreducible fraction $\frac{a}{b}$. So, find the value of $a + b$.

a) 9

b) 10

c) 11

d) 12

Q18.

A horse is tied to the corner of a fence by an 8-meter-long rope that can slide freely. The fence is a rectangle, 6 meters by 9 meters, and keeps the horse on the outside. What is the size, in square meters, of the area through which the horse can move? Use $\pi = 3$.

a) 51

b) 150

c) 151

d) 154

Q19.

Find the largest three-digit number that leaves remainder 1 on division by 3, remainder 2 on division by 4, and remainder 3 on division by 5.

a) 958

b) 798

c) 898

d) 878

Q20.

Let a, b, x, and y be real numbers that satisfy the system of equations below. Accordingly, find the value of $ax^4 + by^4$.

$$\begin{cases} a + b = 24 \\ ax + by = 97 \end{cases}$$

$$\begin{cases} ax^2 + by^2 = 241 \\ ax^3 + by^3 = 823 \end{cases}$$

a) 2464

b) 2357

c) 2269

d) 2174