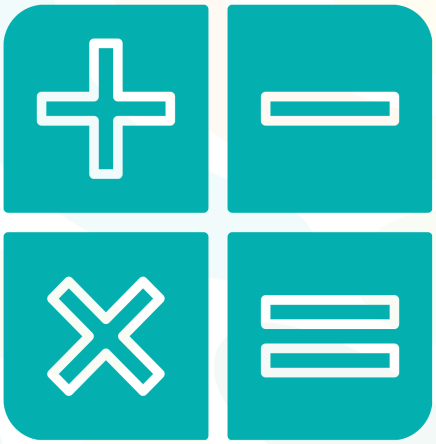
BRAINIACS OLYMPIAD

GRADES 11-12

# MATHEMATICS SAMPLE PAPER

PRACTICAL PART



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**MATHEMATICS SAMPLE PAPER-GLOBAL FINAL**
**Grade: 11-12**
**Time: 120 minutes**
**Total points: 100**

Q1.
A biologist is modeling the growth of a bacterial colony in a petri dish. The area covered by the colony, $A$ (in $cm^2$), after $t$ hours is given by the model:
$A(t) = A_0 \cdot 2^{kt}$
At $t = 0$, the colony covers $1\ cm^2$. After 3 hours, it covers $8\ cm^2$.
Determine the time (in hours) required for the colony to cover exactly $32\ cm^2$.
A) 4 hours
B) 5 hours
C) 6 hours
D) 10 hours

Q2.
An architect is designing a parabolic arch for a monument. On a coordinate grid where the base of the arch lies on the $x$-axis, the arch passes through the points
$(-4,0), (4,0)$
and has a peak height of 8 units at the vertex
$(0,8)$.
A horizontal support beam is to be placed at a height of 6 units above the $x$-axis.
Calculate the width of the arch at the level of this support beam.
A) 4 units
B) 2 units
C) $2\sqrt{2}$ units
D) $4\sqrt{2}$ units

Q3.
A logistics company stacks identical cylindrical pipes in layers. The number of pipes in each layer forms an arithmetic progression.

- The top layer has 5 pipes
- The second layer has 8 pipes
- The third layer has 11 pipes

The pattern continues in the same way.
If the stack contains exactly 10 layers, calculate the total number of pipes in the stack.

A) 320
B) 170
C) 200
D) 185

Q4.
A telecommunications company is placing three signal towers at the coordinates $A(0,0), B(6,0), C(0,8)$ on a map (units in kilometers). They need to identify a point $P(x, y)$ that is equidistant from all three towers in order to install a central hub.
Find the coordinates of point $P$.
A) $(4, 3)$
B) $(2, 3)$
C) $(3, 4)$
D) $(3, 3)$

Q5.
In a quality control experiment, a machine produces components where the probability of a component being defective is $p$. To test a batch, 3 components are selected at random.
The probability that at least one component is defective is $\frac{19}{27}$.
Find the value of $p$.
A) $\frac{1}{3}$
B) $\frac{2}{3}$
C) $\frac{1}{9}$
D) $\frac{4}{9}$

Q6.

A polynomial

$P(x) = x^3 + ax^2 + bx + c$

models the volume of a custom-molded container. The polynomial has roots that form an arithmetic progression.

If the sum of the roots is 9 and the product of the roots is 15, determine the value of the coefficient $b$.

A) 11

B) 23

C) 26

D) 27

Q7.

A surveyor needs to map a triangular plot of land $LMN$.

a) Using a ruler and compass only, construct the triangle $LMN$ where $LM = 10$ cm, $LN = 8$ cm, $\angle MLN = 60°$.

b) Construct the locus of points inside the triangle that are equidistant from the sides $LM$ and $LN$.

c) Construct the locus of points inside the triangle that are exactly $6$ cm from vertex $L$.

d) Shade the region inside the triangle that is closer to $LN$ than to $LM$ and is more than $6$ cm from $L$.

Q8.

A pharmaceutical laboratory tests a new diagnostic kit. Data from 1000 trials shows the following results:

- 95% of patients with the condition test positive *(Sensitivity)*.
- 90% of patients without the condition test negative *(Specificity)*.
- In the general population, 2% of people have the condition.

a) Complete a tree diagram or a contingency table to represent this data.

(Answer:)
Contingency Table

| | Condition | No Condition | Total |
|---|---|---|---|
| Test Positive | 19 | 98 | 117 |
| Test Negative | 1 | 882 | 883 |
| Total | 20 | 980 | 1000 |

b) A person is selected at random from the population and tests positive.
Calculate the probability that the person actually has the condition.
(Answer: $\frac{19}{117}$)

Q9.
A renewable energy plant tracks its power output $W_n$ (in megawatts) over 12 months, where
$n = 1,2,3,\ldots,12.$
The output follows the model
$W_n = 10 + 20\log_2(n)$.
a) On the grid provided *(assume a* $10 \times 10$ *grid)*, sketch the graph of $W_n$ for
$1 \leq n \leq 8.$
b) Calculate the exact total energy output for the first three months using the formula
$\sum_{n=1}^{3} W_n$. (Answer: Sum: $50 + 20\log_2 3$)
c) The plant requires a total output of at least $100$ MW over a two-month period, that is
$W_n + W_{n+1}$.
Determine the smallest value of $n$for which this requirement is satisfied.
(Answer: $n = 4$)

Q10.

A satellite dish is designed so that its cross-section forms a parabola. On a coordinate system, the vertex is at $(0,0)$ and the focus is at $(0,f)$.

The dish receives signals that reflect off its surface, which is modeled by the equation $y = \frac{1}{4f}x^2$.

a) If the dish has a diameter of $20$ cm at its opening and a depth of $5$ cm, find the value of $f$.

(Answer: f=5)

b) A technician places a sensor at the focus. A signal ray travels along the line $x = 4$(Answer: $\left(4, \frac{4}{5}\right)$) and hits the dish. Find the coordinates of the point of impact.

c) Using the properties of the parabola, determine the equation of the tangent line at the point of impact found in part (b).

(Answer: $y = \frac{2}{5}x - \frac{4}{5}$)